Can an embassy issue an apostille?

No, embassies do not issue apostilles.

Only one of the fifty U.S. Secretary of State offices and the United States Department of State in Washington D.C can issue a state or federal apostille certificate.

An embassy can only issue legalization or attestations on a document after it's been first authenticated at the state or federal levels.